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Submitted to the Departments of Naval Architecture and Marine Engineering, and Mechanical Engineering in partial fulfillment of the requirements for the degrees of Naval Engineer, and Master of Science in Mechanical Engineering.

ABSTRACT

Second-order steady wave forces on a restrained, submerged, circular cylinder are studied experimentally as a function of cylinder depth and wave frequency. The experimental results are compared to forces calculated from a linearized potential theory. It is found that in all cases the experimentally observed forces are higher - in some cases by an order of magnitude - and are not attenuated with depth as rapidly as predicted by the theory.

Thesis Supervisor: John Nicholas Newman

Title: Associate Professor of Naval Architecture

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1. INTRODUCTION

Since very early in written history warriors have attempted to escape the sea's surface and utilize submarine warfare. Herodotus (460 B.C.), Aristotle (332 B.C.), and Pliny, the elder (77 A.D.) mention determined attempts to build submersibles. Alexander the Great (356 to 323 B.C.) is the first person known to have descended into the sea in a vessel of any kind.[1]

Since that time submarines have been invented which were propelled by oars, sails, treadles, hand-operated screws, clockwork, springs, steam stored in tubes, chemical engines, compressed air, stored gases, electric motors, and nuclear power. Over three hundred years ago Mother Shipton, a famous English prophetess, predicted the coming of the submarine when writing, "underwater man shall walk, shall ride, shall sleep, shall talk." Her prophesy has come true, but in carrying out her prophesy man has discovered that even though he may be an underwater man in many senses, he has not completely escaped the forces of surface waves.

Man was early to recognize the forces of the sea to be powerful, at times destructive, and inevitable. He set out to study them, attempt to predict them, and to design vessels which would withstand and take advantage of them. Studies of forces on vessels penetrating a free surface are too numerous

to mention here.

The United States Navy commissioned the USS Holland, its first submarine on April 11, 1900. The military importance of submarines has increased at a staggering rate since that time. More recently, commercial and scientific interests have awakened to the vast resources under the sea, hence the use of non-military submersibles has begun to increase at a rate that may exceed that of its military cousin. As more and more use is made of undersea vessels, they become larger -- the largest known maneuverable undersea vessel to date is about ten meters in diameter, greater than one hundred meters in length, and displaces in excess of six thousand tons -- and yet they must be precisely controlled.

In order to be able to provide this precise control, the forces acting on these vessels must be known. One source of such forces is surface waves. A number of studies have been made of the wave induced forces on underwater bodies, but most of these studies have been concerned with an underwater body moving with some translational velocity with respect to the fluid. Examples of these studies are some of Havelock's[2] papers, Kim's [3] paper, and Salvesen's[4] paper.

The present work will concern itself with a body that is not moving, but which is rigidly restrained. Kulin[5] studied the wave forces on submerged cylinders and plates by considering

the effect of a single intumescence which subjects the fluid to acceleration followed by deceleration. He studied (1) the extent local acceleration causes the drag coefficient to depart from steady state values, (2) the extent that friction affects potential flow values of inertial coefficient, and (3) the extent of force dependence upon preceding flow history, as well as on instantaneous conditions. He found that the forces could not be represented by a single universal total resistance coefficient, and that the ratio of the distance of fluid motion to object size was a significant parameter related to vortex formation during fluid deceleration.

In 1962, Ogilvie[6] applied and extended the form of solution that had been developed by Ursell[7] in 1950 to calculate the first-order oscillatory force and the second-order steady force in the following situations: (a) a submerged cylinder is restrained from moving under the effect of incident sinusoidal waves; (b) a submerged cylinder is forced to oscillate sinusoidally, in otherwise calm water; (c) a cylinder, which is neutrally buoyant, is allowed to respond to first-order oscillatory forces. He proves that subject to his assumptions a knowledge of the first-order potential supplies information sufficient to solve these problems. This paper deals with an attempt to experimentally verify his results for case (a), the restained cylinder.

Assume that a circular cylinder is located under a free surface,

(see figure 1.) such that the surface of the cylinder is specified by

$$S(x,y) = x^2 + (y + h)^2 - a^2 = 0. \tag{1}$$

The undisturbed free surface is taken as the x-axis and the instantaneous free surface will be specified by

$$y - Y(x,t) = 0. \tag{2}$$

The required velocity potential, $\Phi(x,y,t)$, satisfies

$$\Phi_{xx} + \Phi_{yy} = 0 \qquad \text{for} \quad y < Y(x,t), \quad \text{and} \quad S(x,y) > 0, \tag{3}$$

$$\Phi_x Y_x - \Phi_y + Y_t = 0 \qquad \text{on} \quad y = Y(x,t), \tag{4}$$

$$gY + \Phi_t + \tfrac{1}{2}(\Phi_x^2 + \Phi_y^2) = 0 \qquad \text{on} \quad y = Y(x,t), \tag{5}$$

and

$$\Phi_x S_x + \Phi_y S_y + S_t = 0 \qquad \text{on} \quad S(x,y) = 0, \tag{6}$$

plus appropriate conditions at infinity. The time average, $\overline{Z^{(2)}(t)}^t$, of the second-order steady force can be shown to be

$$\overline{Z^{(2)}(t)}^t = \overline{X^{(2)}(t)}^t - i\overline{Y^{(2)}(t)}^t$$

$$= -\tfrac{1}{2}ia\rho\int_{-\pi}^{\pi} e^{i\theta}\left[\overline{(\Phi_x)^2}^t + \overline{(\Phi_y)^2}^t\right]_{r=a} d\theta \tag{7}$$

or if we let $\Phi(x,y,t)$ be the real part of a function of

a complex variable, $f(z,t)$, where $z = x + iy$, then

$$\overline{Z^{(2)}(t)}^{t} = -\tfrac{1}{2} i a \rho \int_{-\pi}^{\pi} e^{i\theta} \left[\overline{f'(z,t)\overline{f'(z,t)}}^{t} \right]_{r=a} d\theta \tag{8}$$

$$= -i 2\pi \rho g (HO)\ e^{-2kh}\ Q(ka,kh). \tag{8'}$$

k is the wave number defined as

$$k = \sigma^2/g, \tag{9}$$

and HO is the amplitude of the incident waves.

Define steady vertical force coefficient, β, as

$$\beta = \frac{\overline{Y^{(2)}(t)}^{t}}{2\pi\rho g (HO)^2}. \tag{10}$$

From (8') and (10), it is seen that

$$\beta = e^{-2kh}\ Q(ka,kh). \tag{11}$$

It is this quantity obtained by Ogilvie that will be compared with experimental results in Section IV.

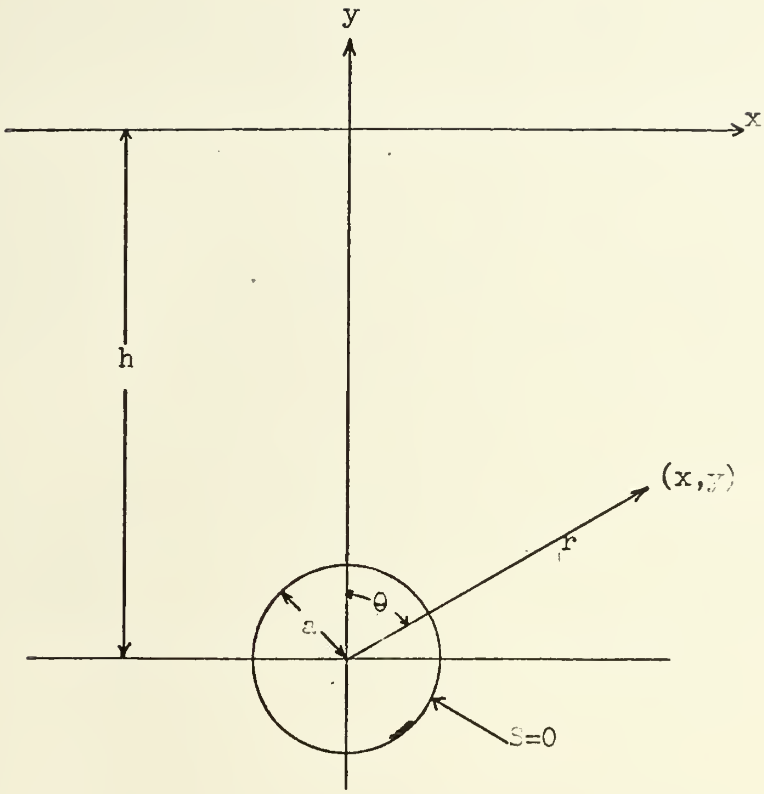


Figure 1. Geometry of the problem

II. TEST APPARATUS AND PROCEDURE

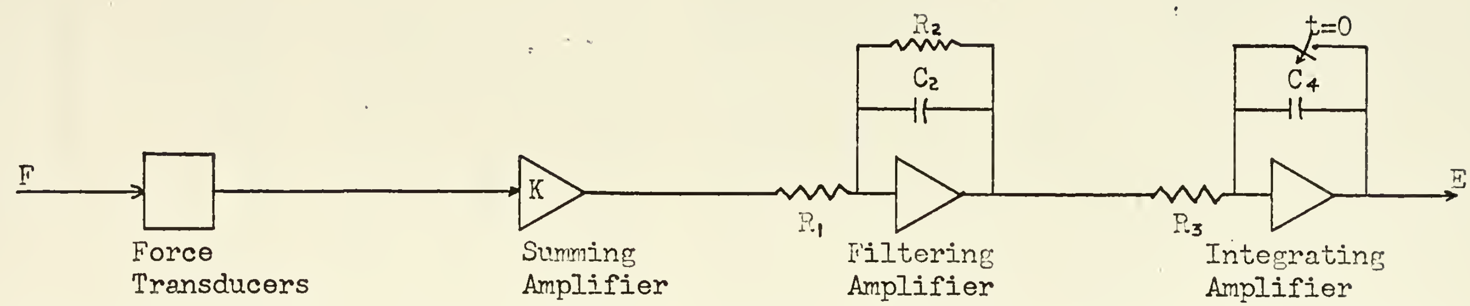
II.A. TEST APPARATUS DESCRIPTION

This experiment was conducted in the wave tank in the M.I.T. Department of Naval Architecture and Marine Engineering Hydrodynamics Laboratory. The working section of the tank is ten feet long, one foot wide, and six inches deep. The wave generator is a vertically oriented pusher, hinged at the bottom, and driven by a push-rod connected off center on a flywheel, which is in turn driven by a variable speed electric motor. The observation section was located about equidistant from the two ends. An energy absorbing beach made of rubberized horse hair was located at the end opposite the wave generator. This beach was not completely successful in preventing reflected waves. This will be considered in Section II.B.

Wave height was measured by a capacitance probe mounted on a carriage which was driven by a variable speed electric motor and ran on tracks along the top of the tank. The output of the probe was fed through a buffer amplifier to a Hewlett Packard Model 7702 B recorder. The carriage was moved at a constant speed before each force observation to obtain data for calculating the reflected to direct wave ratio and then placed at a predesignated point as close as possible to the cylinder to measure wave height during the period of force measurement.

The cylinder itself, a one inch diameter lucite rod 11 15/16 inches long, was suspended below the surface by a 3/32 inch diameter rod tapped into the cylinder at points 1/8 inch from each cylinder end. Any effect on vertical forces by these thin supporting rods was assumed to be negligible. The 1/32 inch clearance between each end of the rod and the tank wall is considered to be smaller than any wall boundary layer thickness, and it is felt that there was no appreciable departure from the two dimensional condition. The tops of the supporting rods were threaded and attached to a yoke which was restrained so that its only degree of freedom was in the vertical direction.

Vertical force on the yoke was measured by two Schaevitz Engineering Model No. FTA 3 Force Transducers, one on each end. The electrical outputs of these transducers were connected and processed as shown schematically in figure 2. As seen from the figure the output voltage was directly proportional to the time integral of the steady force component, a.



F = Force in
= a + b SIN σt

E = Voltage out

$$\frac{E}{K} = \frac{aT_2}{T_1 T_3} t + \frac{b}{T_1 T_3}\left(\frac{1}{\sigma^2 + (1/T_2^2)}\right)\left[\frac{1}{\sigma T_2}(1 - \cos \sigma t) - \sin \sigma t\right]$$

$T_1 = R_1 C_1 = 0.8$ sec. $\qquad T_2 = R_2 C_2 = 2.0$ sec

$T_3 = R_3 C_4 = 1.0$ sec $\qquad \sigma > 10$ rad/sec

Therefore $\qquad E \approx Kat$

Figure 2. Functional force measurement circuit

II.B. PROCEDURE

As mentioned in Section II.A., the energy absorbing beach was not completely successful in preventing reflected waves and it was considered necessary to take these reflected waves into account. A method of doing this by the use of a standing wave ratio, p, is developed in Appendix A. That result is given in (A7) and is repeated here.

$$\beta = \frac{\beta 0}{(1 + p^2)} \qquad (12)$$

The values of HX and HM used in (A8) were obtained by allowing the capacitance probe carriage to move at constant speed along the tracks as mentioned in Section II.A. and observing the maximum and minimum values of wave height recorded.

The wave height recorder was calibrated by changing the still water level by a known amount and observing the change in recorder level. Because of the sensitivity of a capacitance probe, the calibration was repeated frequently and the appropriate calibration factor was used in each calculation of β.

Force measurement calibration was accomplished by first opening the shorting switch shown bypassing the integrating amplifier in figure 2. for a known period of time, TD, and observing the integrated drift voltage, ED. The switch was then closed and a known weight, Y, was placed on the cylinder supporting

yoke. The switch was reopened for a known period of time, TS, and the integrated signal voltage, ES, was observed. The calibration factor, AE, was then determined by the following formula:

$$AE = Y/E \qquad (13)$$

where

$$E = (ES/TS) - (ED/TD). \qquad (14)$$

This factor was found to remain constant at 0.036 pounds per volt.

The unknown steady wave force, $\overline{Y^{(2)}(t)}^{t}$, was found by the following equation, using values of ES, TS, ED, TD, HM, and HX observed for various cylinder depths, h, and incident wave frequencies, F.

$$\overline{Y^{(2)}(t)}^{t} = (E)(AE) \qquad (15)$$

$\beta 0$ was calculated using (10) and (15) and β was calculated by (12).

Calculations were performed at the M.I.T. Computer Center using the program in Appendix B.

III. RESULTS

Experimental data is tabulated in Appendix C. Values of β calculated from this data using (12) are tabulated on the following page.

These values of β are presented graphically in figures 3. through 9. along with curves for ka = 0.2 and 0.5 from Ogilvie's theory.

In drawing the experimental β curves, curves for the next higher and lower values of ka were considered as well as β values for the curve being drawn. As a result the experimental curves for some ka values do not pass through all data points.

The individual β curves from figures 3. through 9. are reproduced in figure 10. for ease of comparison.

2KH	KA	BETA
0.75	0.25	0.2939
1.00	0.25	0.1746
1.50	0.25	0.0997
2.50	0.25	0.0195
3.50	0.25	0.0154
4.50	0.25	0.0020
0.90	0.30	0.3499
1.20	0.30	0.1912
1.80	0.30	0.0674
3.00	0.30	0.0259
4.20	0.30	0.0076
5.40	0.30	0.0009
1.05	0.35	0.4442
1.39	0.35	0.2148
2.09	0.35	0.0696
3.49	0.35	0.0261
4.88	0.35	0.0093
6.28	0.35	0.0029
1.20	0.40	0.4172
1.60	0.40	0.2186
2.40	0.40	0.0918
4.01	0.40	0.0311
5.61	0.40	0.0093
7.21	0.40	0.0032
1.35	0.45	0.4452
1.79	0.45	0.2006
2.69	0.45	0.0999
4.48	0.45	0.0289
6.28	0.45	0.0094
8.07	0.45	0.0007
1.51	0.50	0.5359
2.01	0.50	0.2533
3.01	0.50	0.1018
5.02	0.50	0.0253
7.03	0.50	0.0047
9.03	0.50	0.0013
1.65	0.55	0.5429
2.20	0.55	0.2610
3.31	0.55	0.1103
5.51	0.55	0.0151
7.72	0.55	0.0028
9.92	0.55	0.0013

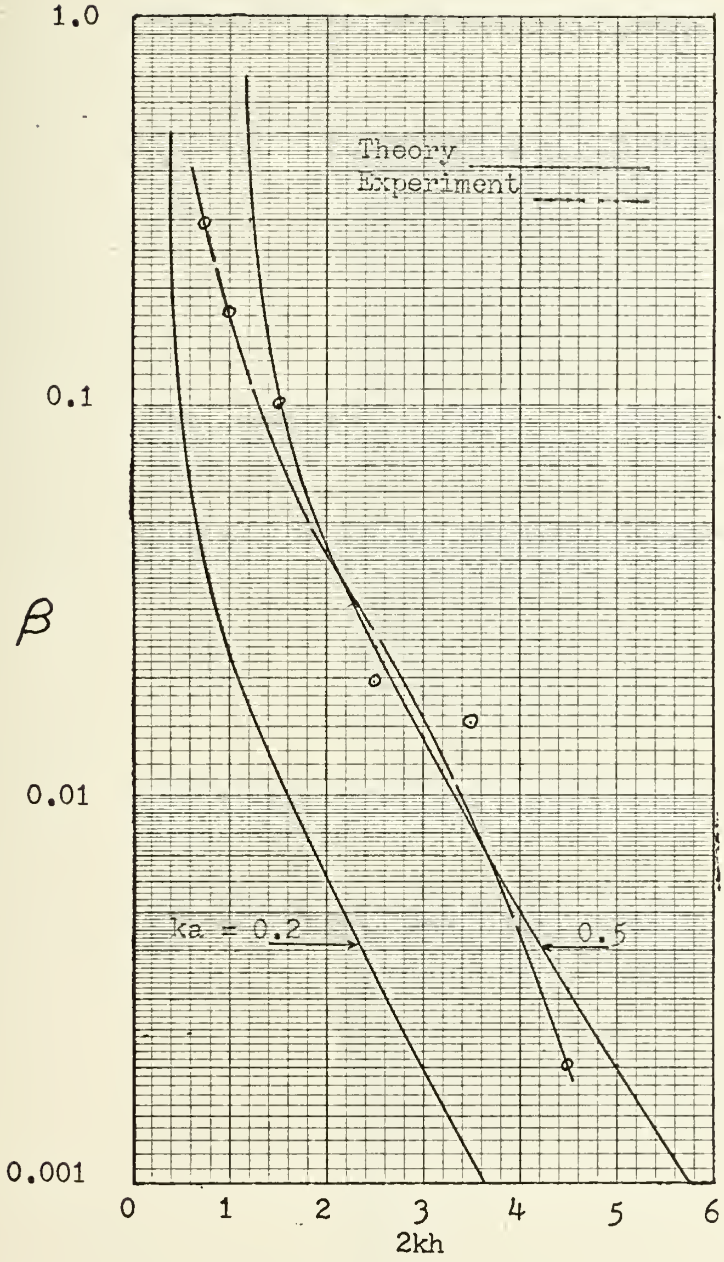


Figure 3. Steady vertical force coefficient, ka = 0.25

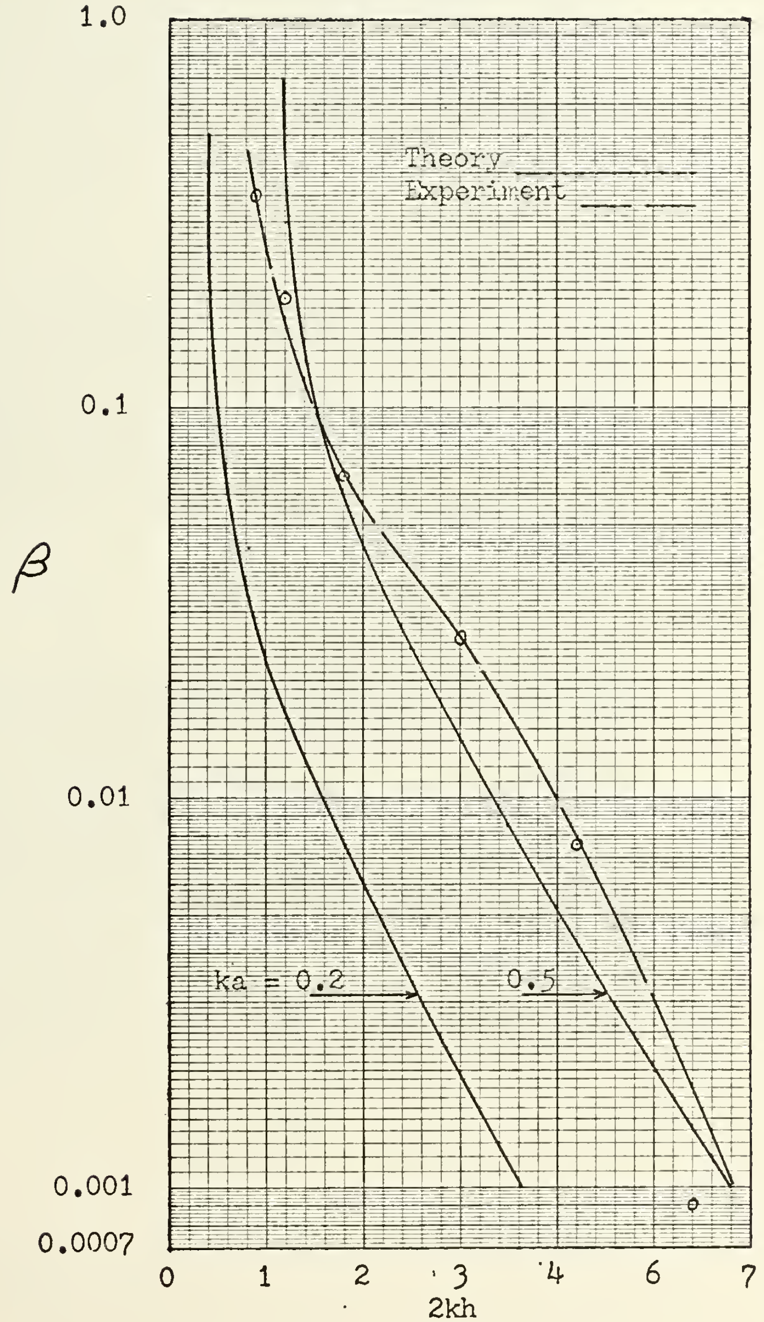


Figure 4. Steady vertical force coefficient, ka = 0.3

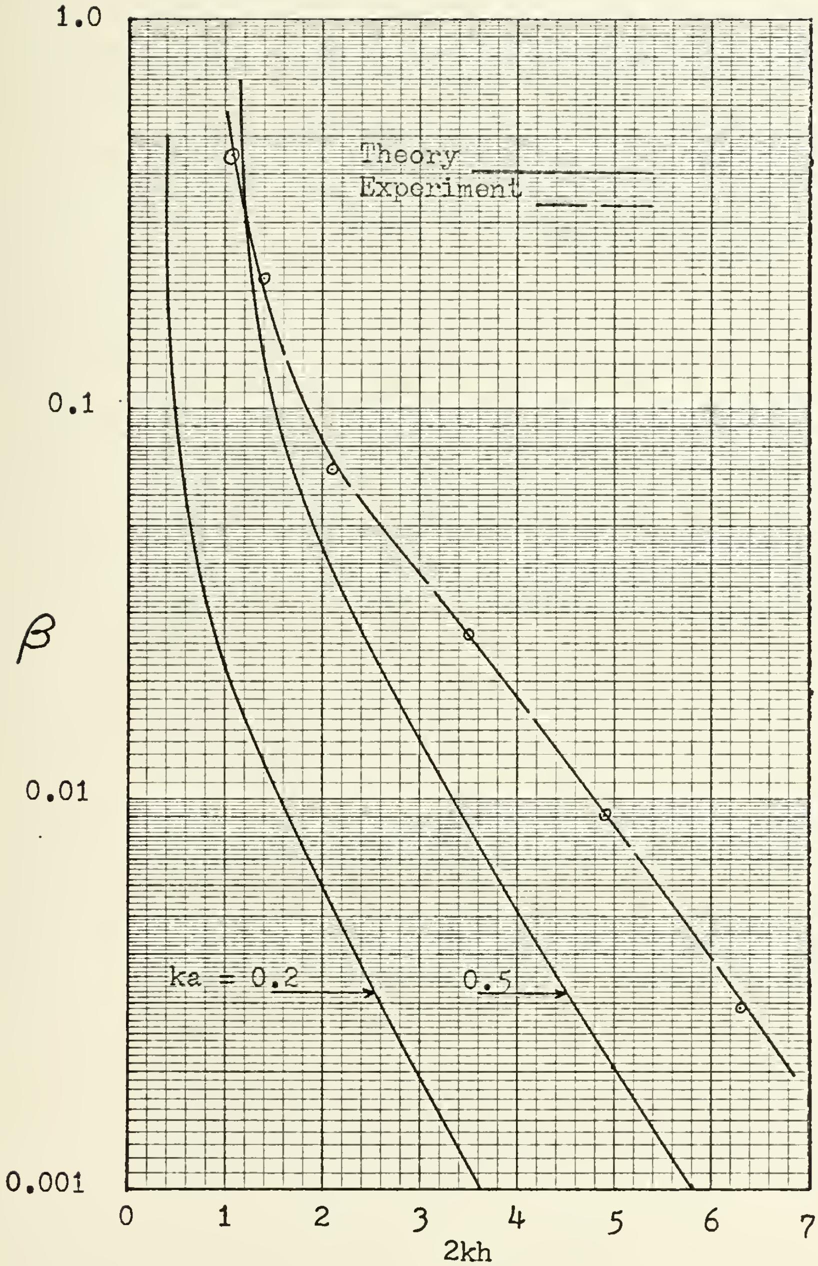


Figure 5. Steady vertical force coefficient, ka = 0.35

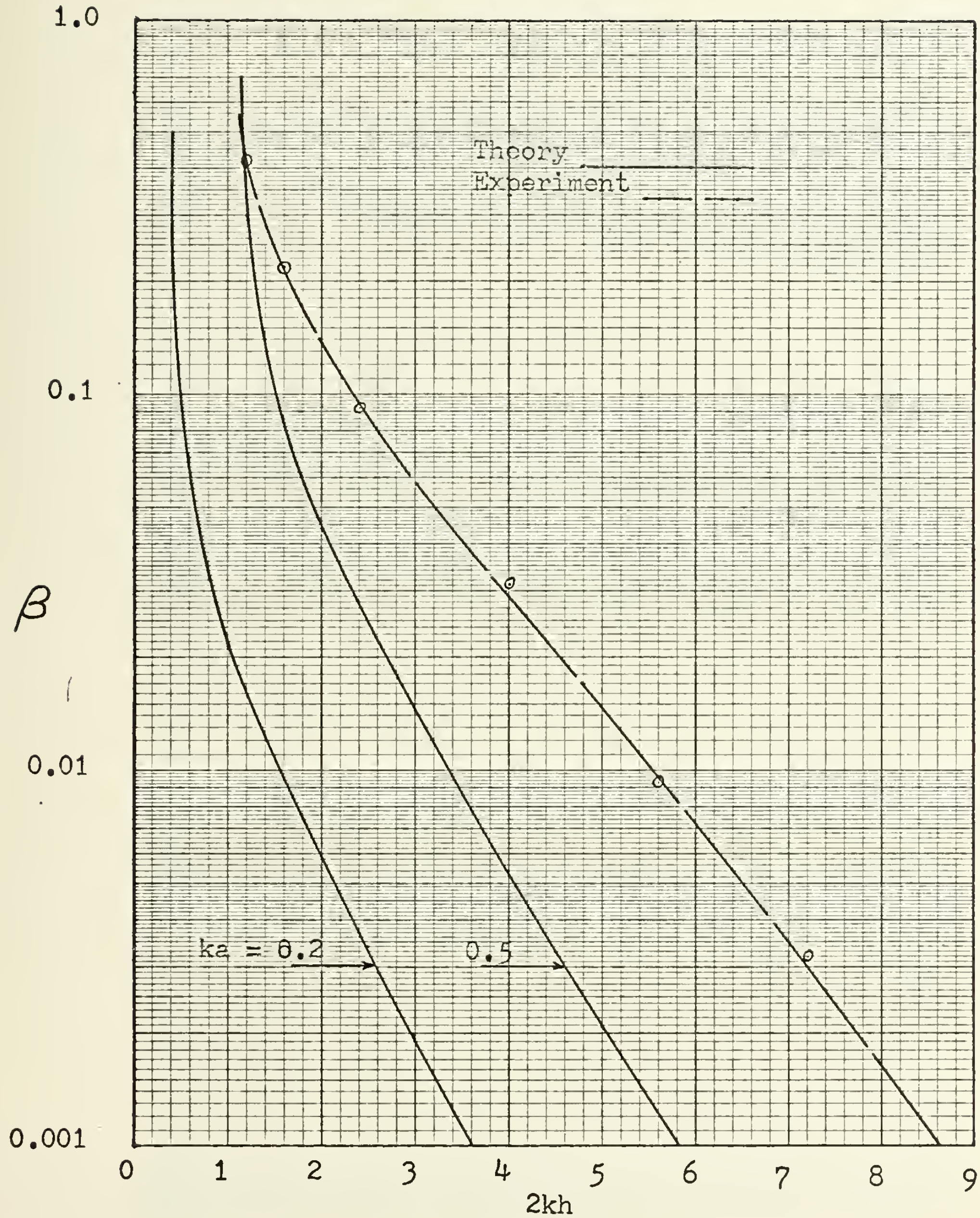


Figure 6. Steady vertical force coefficient, ka = 0.4

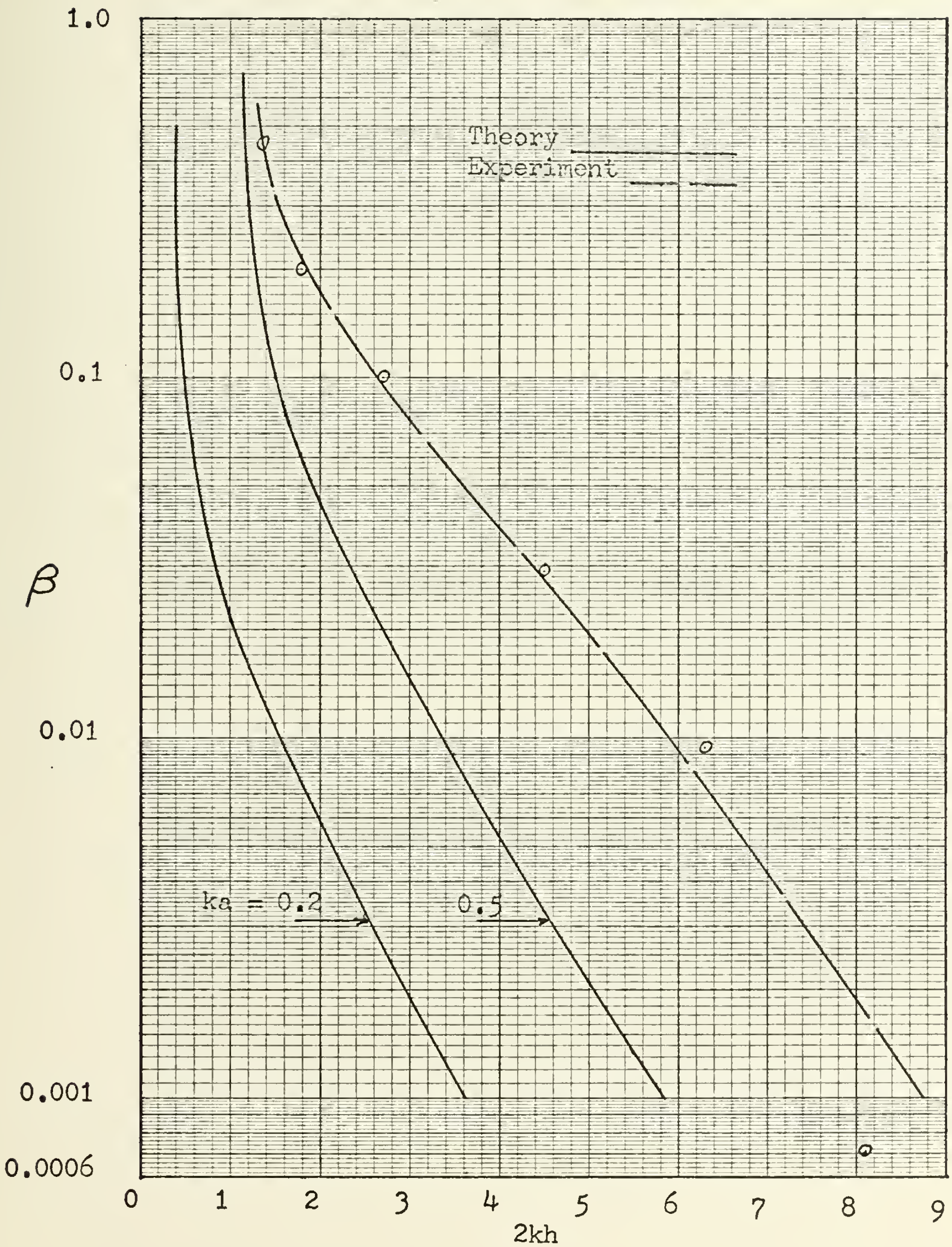


Figure 7. Steady vertical force coefficient, ka = 0.45

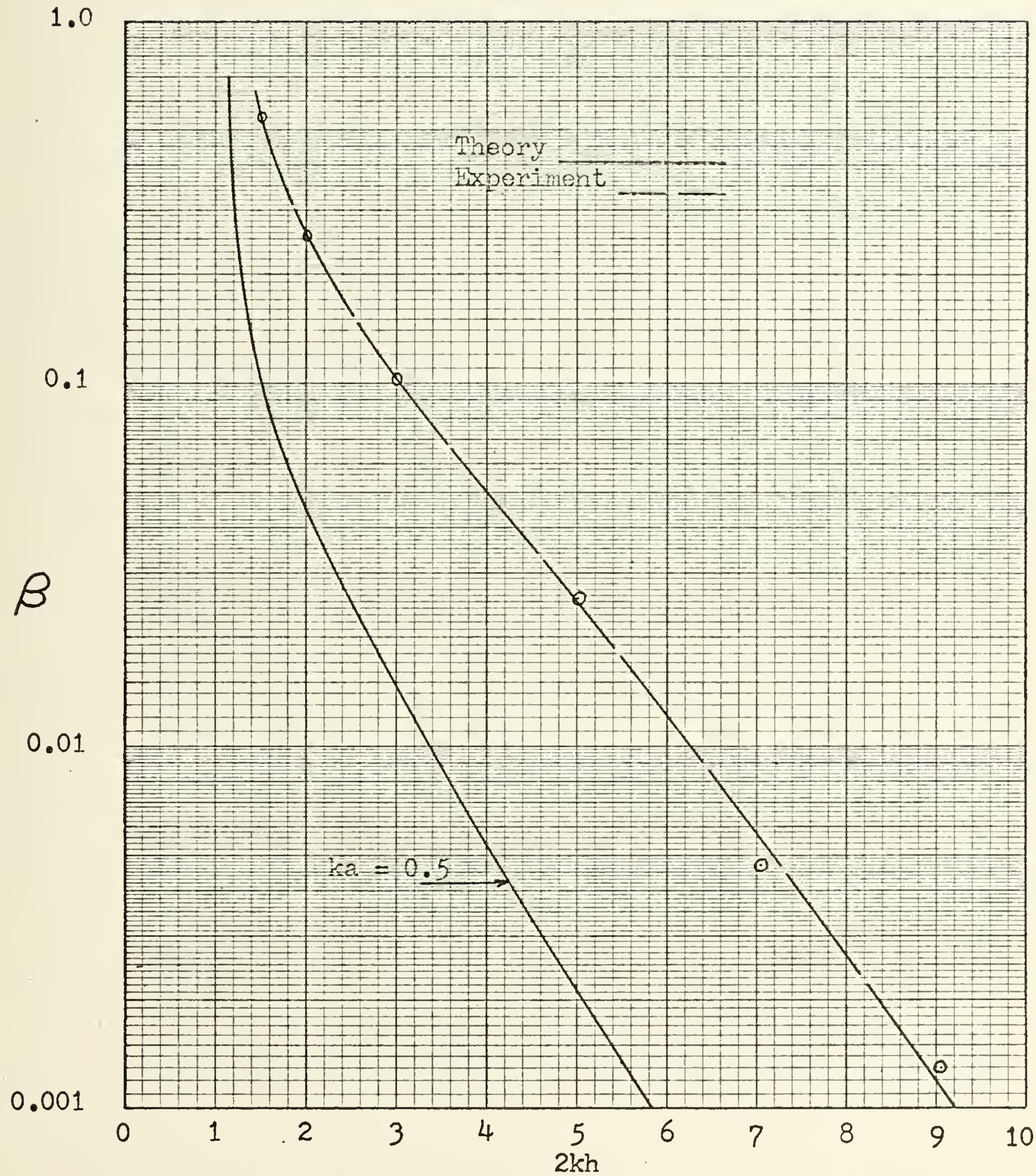


Figure 8. Steady vertical force coefficient, ka = 0.5

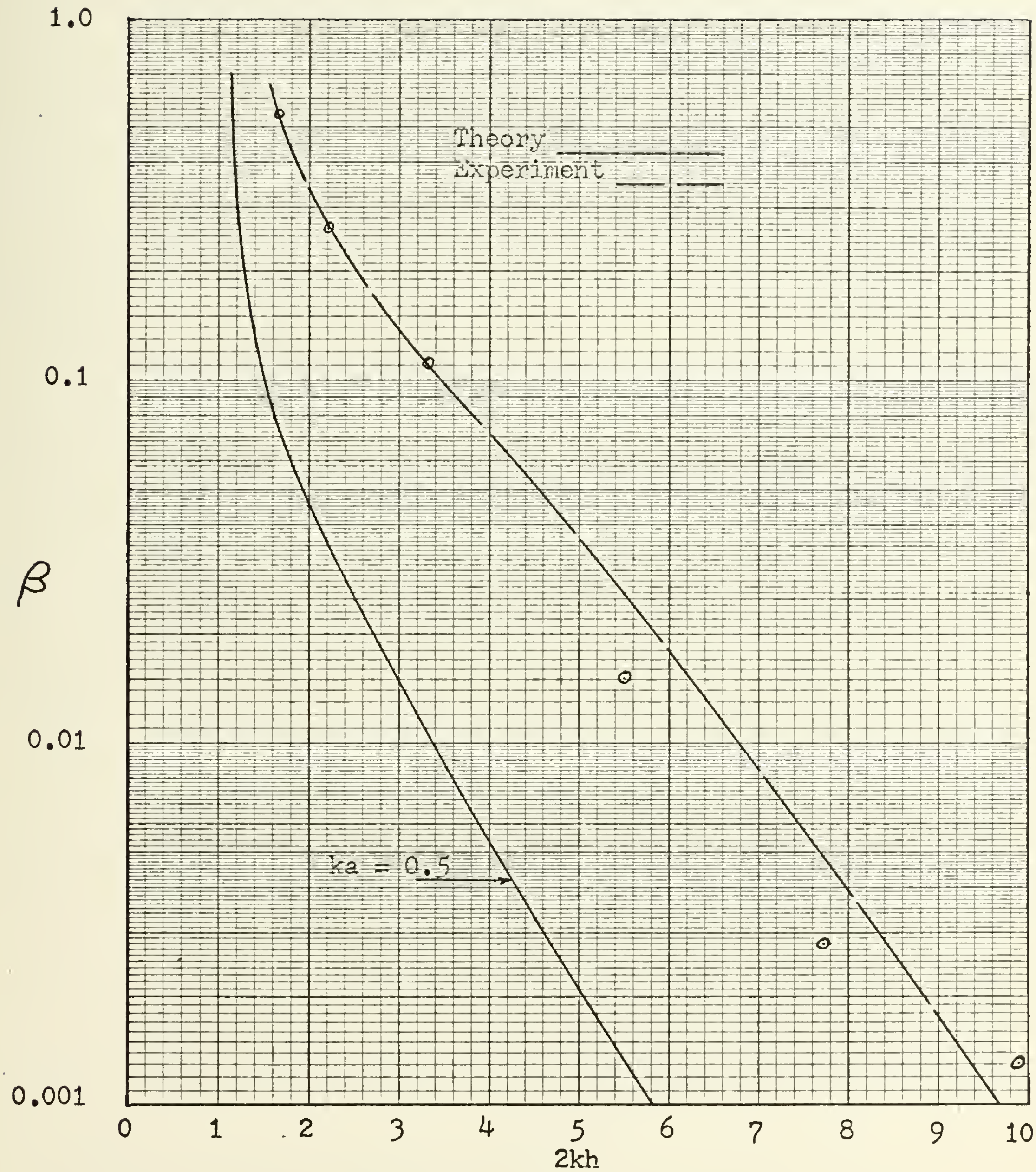


Figure 9. Steady vertical force coefficient, ka = 0.55

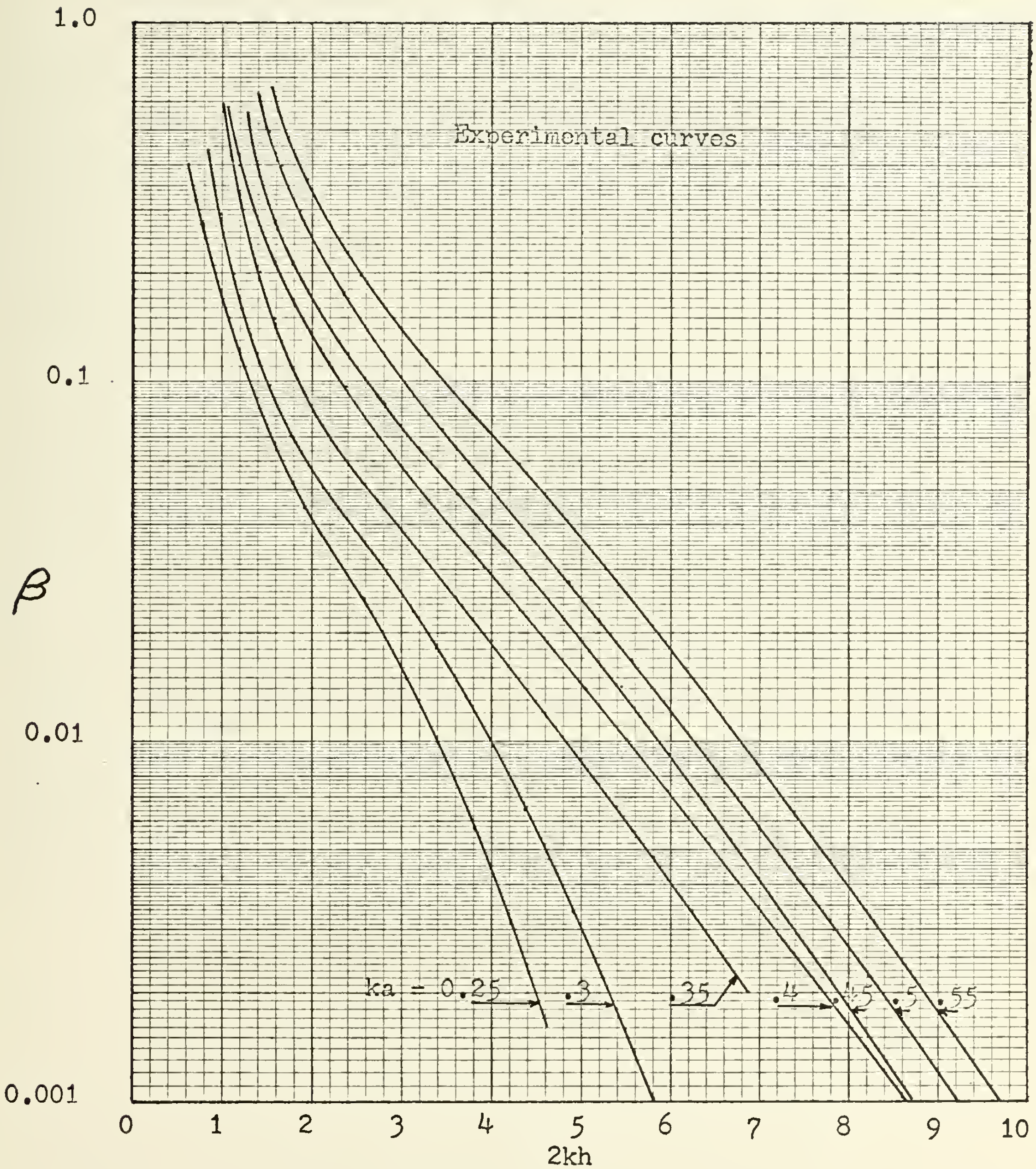


Figure 10. Steady vertical force coefficients

IV. DISCUSSION OF RESULTS

Reference to figures 3. through 9. shows that the experimental curves of steady vertical force coefficient are in all cases higher -- in some cases more than an order of magnitude -- than the theoretical curves, and in all cases except for ka = 0.25 and 0.3 the slope is less steep than the theoretical curves.

It is also noted that the experimental curves all have inflection points between the third and fourth data points -- at an h/a ratio of about 4. From figure 10. it is easily seen that this inflection is much more pronounced in the curves for smaller values of ka -- longer wave lengths. Since for the longest wavelength investigated the tank depth was over a half wave length it was expected that it would have an infinite effective depth; however, a finite depth effect seems to be a plausible explanation for these curve inflections.

V. CONCLUSIONS AND RECOMMENDATIONS

From Sections III and IV it may be concluded that Ogilvie's potential theory does predict the general behavior of the steady vertical force coefficient, but that in all cases it predicts a force smaller than that measured experimentally and that the forces will be attenuated more rapidly with depth than that experimentally observed.

Because of the curve inflections noted in Section IV and since smaller values of ka are more likely to be encountered at sea, i.e., a one hundred meter wave length and a ten meter diameter cylinder give a ka of 0.314 -- most waves of interest are longer and most diameters are smaller, hence most ka's of interest are smaller, it is felt that an extension of this experiment to smaller values of ka in facility with an infinite effective depth and no reflected waves would be rewarding.

APPENDIX A

Theory Modification to Account for Reflected Waves

Using the 'circle theorem' of Milne-Thompson[8], the complex potential function, f(z,t), in (8) can be represented by

$$f_1(z,t) = A\left[e^{-i(\sigma t + kz)} + e^{-kh} e^{i\left[\sigma t + \frac{ka^2}{(z + ih)}\right]}\right] \tag{A1}$$

where

$$A = (HO)g/\sigma. \tag{A2}$$

Using (A1), (8) and (10), is found to be

$$\beta = e^{-2kh} (ka)\, I_1(2ka). \tag{A3}$$

The complex potential function for a reflected wave traveling in the opposite direction with an unknown phase angle, Ψ, and a magnitude of p times the direct wave can be given by

$$f_2(z,t) = pAe^{i\Psi}\left[e^{-i(\sigma t - k\bar{z})} + e^{-kh} e^{i\left[\sigma t - \frac{ka^2}{(\bar{z} - ih)}\right]}\right]. \tag{A4}$$

Now the complex potential function for a direct and reflected wave existing simultaneously is given by

$$f(z,t) = f_1(z,t) + f_2(z,t). \tag{A5}$$

Using (A5), (8) and (10), βO is found to be

$$\beta O = (1 + p^2)\, e^{-2kh} (ka)\, I_1(2ka). \tag{A6}$$

Consequently

$$\beta = \beta 0/(1 + p^2) \tag{A7}$$

where β would be the steady vertical force coefficient if there were no reflected wave present, $\beta 0$ is the steady vertical force coefficient with a reflected wave present, p is the standing wave ratio given by

$$p = \frac{HX - HM}{HX + HM} \tag{A8}$$

and HX and HM are the maximum and minimum wave amplitudes that exist as a result of the summation of the direct and reflected waves. HO used in (A2) is the average wave amplitude given by

$$HO = \frac{HX + HM}{2}. \tag{A9}$$

The complex potential functions given by (A1) and (A4) do not satisfy the free surface conditions (4) and (5), hence (A3) and (A6) can be expected to give reasonable results only if $ka \ll 1$ and/or $2kh \gg 1$. However, it is assumed that (A7) is valid throughout the interest range of ka and kh values.

APPENDIX B

Computer Program for Steady Vertical Force Coefficient Calculation

```
      REAL K,KA
      READ(5,100)A,AE
100   FORMAT(2F10.0)
      WRITE(6,999)
999   FORMAT(////)
      WRITE(6,101)
101   FORMAT(11X,'2KH',4X,'KA',3X,'BETA')
1     READ(5,103)H,F,ED,TD,ES,TS,HX,HM,AH
103   FORMAT(8F5.0,F6.0)
      IF(F.GT.999.) GO TO 2
      K  =.0000284*(F**2)
      KA=K*A
      TWOKH=2.*K*H
      E=(ES/TS)-(ED/TD)
      Y=AE*E
      HO=AH*(HX+HM)/4.
      BETAO=.367*Y/(HO**2)
      P  =(HX-HM)/(HX+HM)
      BETA=BETAO/(1.+(P**2))
      WRITE(6,102) TWOKH,KA,BETA
102   FORMAT(F14.2,F6.2,F8.4)
      GO TO 1
2     WRITE(6,500)
500   FORMAT(1H1)
      CALL EXIT
      END
```

APPENDIX C

Experimental Data Tabulation

A (IN.)	AE (LBS/VOLT)
0.5	0.036

H (IN)	FREQ (CPM)	ED (VOLT)	TD (SEC)	ES (VOLT)	TS (SEC)	HX (DIV)	HM (DIV)	AH (IN/DIV)
0.75	132.7	-0.670	300.	4.410	20.	20.0	17.5	0.011
1.00	132.7	0.848	390.	3.890	30.	20.2	16.5	0.011
1.50	132.7	0.262	180.	2.580	30.	22.0	17.5	0.011
2.50	132.7	0.676	150.	1.250	60.	21.5	17.8	0.011
3.50	132.7	-0.645	300.	0.450	60.	21.5	17.0	0.009
4.50	132.7	0.912	180.	0.382	60.	21.5	17.5	0.009
0.75	145.3	0.062	30.	4.380	15.	21.0	18.2	0.011
1.00	145.3	0.085	30.	4.770	30.	20.0	19.0	0.011
1.50	145.3	0.162	180.	1.860	30.	22.0	19.0	0.011
2.50	145.3	0.150	30.	1.540	60.	20.5	18.0	0.011
3.50	145.3	-0.461	120.	0.165	120.	21.5	19.0	0.009
4.50	145.3	0.912	180.	0.341	60.	22.0	19.5	0.009
0.75	156.7	0.189	30.	4.670	15.	19.0	16.7	0.011
1.00	156.7	0.105	30.	4.870	30.	20.0	17.0	0.011
1.50	156.7	0.062	180.	2.220	30.	25.0	19.0	0.011
2.50	156.7	0.165	30.	1.680	60.	21.0	19.0	0.011
3.50	156.7	-0.200	60.	0.289	120.	20.5	18.0	0.009
4.50	156.7	0.030	30.	0.164	60.	20.5	17.0	0.009
0.75	168.0	0.316	30.	4.390	15.	18.0	17.5	0.011
1.00	168.0	0.125	30.	4.820	30.	19.0	17.5	0.011
1.50	168.0	-0.038	180.	1.960	30.	19.4	17.0	0.011
2.50	168.0	0.170	30.	1.640	60.	19.0	17.0	0.011
3.50	168.0	-0.876	90.	-0.232	60.	20.0	19.0	0.009
4.50	168.0	0.012	30.	0.141	60.	20.0	18.5	0.009
0.75	177.7	0.443	30.	4.880	15.	19.0	17.0	0.011
1.00	177.7	0.145	30.	4.810	30.	19.5	18.5	0.011
1.50	177.7	-0.138	180.	2.300	30.	19.5	18.5	0.011
2.50	177.7	0.185	30.	1.610	60.	19.0	17.5	0.011
3.50	177.7	-0.876	90.	-0.278	60.	19.0	17.0	0.009
4.50	177.7	-0.071	30.	-0.116	60.	20.0	18.0	0.009
0.75	188.0	0.570	30.	4.850	15.	17.0	15.5	0.011
1.00	188.0	0.162	30.	5.040	30.	18.5	16.0	0.011
1.50	188.0	-0.238	180.	1.970	30.	18.0	17.0	0.011
2.50	188.0	0.200	30.	1.370	60.	18.0	16.5	0.011
3.50	188.0	-0.045	90.	0.327	165.	18.5	17.0	0.009
4.50	188.0	-0.883	300.	-0.136	60.	18.0	17.5	0.009
0.75	197.0	6.970	300.	4.500	15.	16.0	14.8	0.011
1.00	197.0	0.909	150.	4.360	30.	16.5	15.0	0.011
1.50	197.0	-0.284	240.	1.730	30.	16.5	15.0	0.011
2.50	197.0	0.672	90.	0.963	60.	17.0	15.5	0.011
3.50	197.0	0.749	120.	0.955	120.	19.5	19.0	0.009
4.50	197.0	-0.883	300.	-0.127	60.	20.0	18.5	0.009

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